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ASTRON TYPE EQUILIBRIUM IN THE ABSENCE OF AN APPLIED MAGNETIC F--ETC(U)

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Astron Type Equilibrium in the Absence of an Applied Magnetic Field

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NAVAL RESEARCH LABORATORY
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It is shown that the Vlasov-Maxwell equations predict the existence of rotating layer equilibria (rigid rotor) in the absence of an external magnetic field, provided that the density of the layer exceeds a critical density.

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ASTRON TYPE EQUILIBRIUM IN THE ABSENCE OF AN APPLIED MAGNETIC FIELD

Using a cold fluid model, Yoshikawa[1] has shown the existence of equilibrium for a space charge neutralized, long rotating electron beam propagating in the absence of an externally applied magnetic field. In Yoshikawa's calculation the effect of the conducting wall surrounding the beam has been neglected. In recent experiments[2] at the Naval Research Laboratory, a rotating, overdense, space charge neutral relativistic electron beam has successfully propagated inside a conducting tube in the absence of an external magnetic field. In both the above two cases the azimuthal, self magnetic field (B_θ^s) of the propagating beam plays a significant role in the existance of equilibrium.

In this note, we make use of the steady state ($\frac{\partial}{\partial t} = 0$) Vlasov-Maxwell equations to show the existence of rotating layer (Astron type) equilibrium in the absence of an external magnetic field.

The azimuthally symmetric, z-independent equilibrium configuration is shown schematically in Fig. 1. It is assumed that the layer is space charge neutralized, its axial velocity $v_z = 0$ and the magnetic flux inside the conducting tube is conserved. The nonrelativistic, rotating layer is described by the distribution function

$$f_i^o(H-\omega P_\theta) = (m\bar{n}/2\pi)\,\delta(H-\omega P_\theta - k_1), \tag{1}$$

where $m, \omega, \bar{n}$ and k_1 are constants. In addition, the total energy H

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and the canonical angular momentum P_θ are constants of the motion. In the presence of an externally applied magnetic field, the equilibrium properties of proton (P)-layers that are described by the distribution function of Eq. (1) have been studies by Kapetanakos et al and their stability properties at a frequency near ω by Uhm and Davidson.[4]

Since $v_z = 0$, the argument of delta function in Eq. (3) may be expressed, in the nonrelativistic case, as

$$H-\omega P_\theta - k_1 = (m/2)[(v_\theta - r\omega)^2 + v_r^2] + U(r), \qquad (2)$$

where

$$U(r) = -(qr/c)A_\theta^o(r)\omega - mr^2\omega^2/2 - k_1, \qquad (3)$$

and A_θ^o is the equilibrium magnetic vector potential that describes the axial, self magnetic field $B_z^s(r)$ of the rotating layer.

It is easy to show from the results of ref. 3, that in the absence of an external magnetic field B_o, the density profile $n^o(r)$, the azimuthal current density $J_\theta^o(r)$, the mean azimuthal velocity of the rotating layer $V_\theta^o(r)$ the self magnetic field $B_z^s(r)$ and the inner a_1 and outer a_2 radii are given by

$$n^o(r) = \begin{cases} 0, & 0 \le r < a_1, \\ \bar{n}, & a_1 \le r \le a_2, \\ 0, & a_2 < r < b, \end{cases} \qquad (4)$$

$$J_\theta^o(r) = \begin{cases} 0, & 0 \le r < a_1, \\ q\bar{n}V_\theta^o(r) & a_1 \le r \le a_2, \\ 0, & a_2 < r \le b, \end{cases} \qquad (5)$$

$$V_\theta^o(r) = \langle v_\theta(r) \rangle = (1/\bar{n}) \int v_\theta f_i^o \, d^2v = \omega r, \quad a_1 \le r \le a_2, \tag{6}$$

$$B_z^s(r) = \begin{cases} \dfrac{4\pi I_\ell}{c} \left[1 - \dfrac{(a_1^2 + a_2^2)}{2b^2} \right], & 0 \le r < a_1, \\[2ex] \dfrac{4\pi}{c} I_\ell \left[\dfrac{(a_2^2 - r^2)}{(a_2^2 - a_1^2)} - \dfrac{(a_1^2 + a_2^2)}{2b^2} \right], & a_1 \le r \le a_2, \\[2ex] \dfrac{-4\pi}{c} I_\ell \left[\dfrac{(a_1^2 + a_2^2)}{2b^2} \right], & a_2 < r \le b, \end{cases} \tag{7}$$

and

$$a_1^2 = - \frac{16 k_1 \lambda^{*2}/(m\omega^2/b^2)}{1 \pm \left\{ 1 + 16(\lambda^*/b)^2 \left[1 + 4k_1/(m\omega^2 b^2) \right] \right\}^{\frac{1}{2}}} = - \frac{k_1 c^2}{\pi q I_\ell \omega}, \tag{8}$$

$$a_2^2 = b^2 + \frac{8\lambda^{*2} \left[1 + 2k_1/(m\omega^2 b^2) \right]}{1 \pm \left\{ 1 + 16(\lambda^*/b)^2 \left[1 + 4k_1/(m\omega^2 b^2) \right] \right\}^{\frac{1}{2}}} =$$

$$b^2 + \frac{mc^2 \omega b^2 \left[1 + 2k_1/(m\omega^2 b^2) \right]}{2\pi q I_\ell}. \tag{9}$$

In these equations $I_\ell = (q\bar{n}\omega/2)(a_2^2 - a_1^2)$ is the azimuthal current per unit length, $\lambda^* = c/\omega_p$, $\omega_p^2 = 4\pi\bar{n}q^2/m$ and k_1, I_ℓ and ω are negative for $q > 0$. Since $a_2^2 \le b^2$ and $(a_2^2 - a_1^2)$ should be real, Eqs. (8) and (9) give

$$\frac{1}{\frac{1}{2}\left[1 + \dfrac{b^2}{16\lambda^{*2}} \right]} \le \left(\frac{\omega}{\omega_o} \right)^2 \le 1, \tag{10}$$

where $\omega_o^2 = 2|k_1|/mb^2$.

According to Eq. (10), radially confined equilibria $(a_2 \leq b)$ exist only for

$$b \geq 4\lambda^*, \qquad (11)$$

i.e., for $\bar{n} = \bar{n}_{min} \geq 4/\pi b^2 R_o)$, where R_o is the charged particle classical radius. For a rotating electron layer inside a conducting tube of radius $b = 4$ cm, Eq. (11) predicts that equilibrium exists only for densities above $3 \times 10^{11} cm^{-3}$. For protons the minimum density is higher by the mass ratio m_p/m_e.

At the minimum density $(\bar{n}_{min} = 4/\pi b^2 R_o)$, $\omega^2 = \omega_o^2$ and the inner and outer radii of the layer are $a_1 = b/\sqrt{2}$ and $a_2 = b$. The allowed values of $(\omega/\omega_o)^2$ as a function of $b/4\lambda^*$ are shown in Fig. 2.

For $q > 0$, the function $U(r)$ of Eq. (3) is negative and has the property $U(a_1) = U(a_2) = 0$. The peak of the envelop function $U(r)$ occurs at a radius that can be determined from

$$\left.\frac{dU(r)}{dr}\right|_{r = \rho} = 0,$$

and is equal to

$$\rho^2 = (a_1^2 + a_2^2)/2. \qquad (12)$$

Using Eqs. (7) and (11), it can be shown that

$$\omega = - \Omega(\rho) = - \frac{qB_z^s(\rho)}{mc} . \qquad (13)$$

The magnetic field becomes equal to zero at the radius ρ_o, that is easily determined from Eq. (7). If ν $[= \frac{\pi \bar{n} q^2}{mc^2} (a_2^2 - a_1^2)]$ is the layer strength (Budker) parameter, Eq. (7) gives

$$\frac{(a_2^2-\rho_2^2)}{(a_2^2-a_1^2)} = \frac{(\nu+1)}{2\nu} \tag{14}$$

Since $I_\ell = \nu\omega mc^2/2\pi q$, the inner a_1 and outer a_2 radii can be written as

$$\frac{a_1^2}{b^2} = \left(\frac{\omega_o}{\omega}\right)^2 \frac{1}{\nu}, \tag{15}$$

and

$$\frac{a_2^2}{b^2} = (1+\frac{1}{\nu}) - \frac{a_1^2}{b^2}, \tag{16}$$

substituting Eqs. (15) and (16) in Eq. (12), it is obtained

$$\frac{\rho^2}{b^2} = \frac{1}{2}(1+\frac{1}{\nu}) \quad . \tag{17}$$

For $(\omega/\omega_o) = 1$, Eqs. (15) and (16) give $(a_2/b)^2 = 1$ and

$$(a_1/b)^2 = \frac{1}{2} \pm \frac{1}{2}(1-\bar{n}_{min}/\bar{n})^{\frac{1}{2}}. \tag{18}$$

When $\bar{n} >> \bar{n}_{min}$, the only physically meaningful root of Eq. (18) is $(a_1/b)^2 = 0$, i.e., the layer becomes solid with a radius equal to the radius of the surrounding conducting tube.

In addition, for $\bar{n} >> \bar{n}_{min}$, Eq. (10) gives for the minimum allowed frequency of rotation ω

$$\left(\frac{\omega}{\omega_o}\right)^2 \simeq 2\bar{n}_{min}/\bar{n} \quad . \tag{19}$$

Substituting Eq. (19) into Eqs. (15) and (16), we get $\frac{a_1}{b} = \frac{1}{2}$ and $\frac{a_2}{b} = \frac{\sqrt{3}}{2}$.

For $b \to \infty$, $\omega_o \to 0$ and since $(a_2^2-\rho^2)/(a_2^2-a_1^2) = \frac{1}{2}$, the frequency of rotation ω remains $\neq 0$ for $I_\ell \neq 0$, as may be seen from Eqs. (7) and (13). Thus, according to Eq. (10), equilibrium does not exist.

In summary, I have shown that the Vlasov-Maxwell equations predict the existence of rigid-rotor type equilibria in the absence of an external magnetic field, provided that the particle density in the layer exceeds a minimum density given by $\bar{n}_{min} = 4/\pi b^2/R_o^2$. For a proton layer confined inside a tube of b = 10 cm, $\bar{n}_{min} = 8 \times 10^{13} cm^{-3}$, which can be easily obtained with existing pulsed proton sources.[5] However, for times longer than the ion-electron collision time the situation becomes more complicated because of the possibility of excitation of electron counter current.

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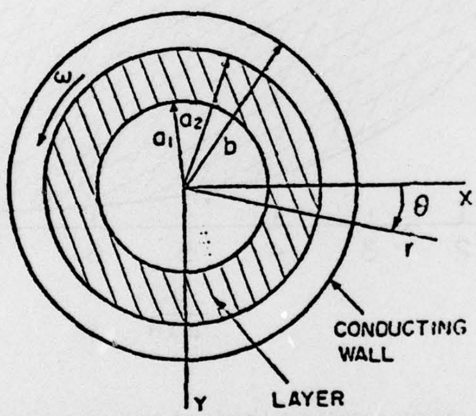

Fig. 1 — Equilibrium configurations of a long, rotating, space-charge neutral layer

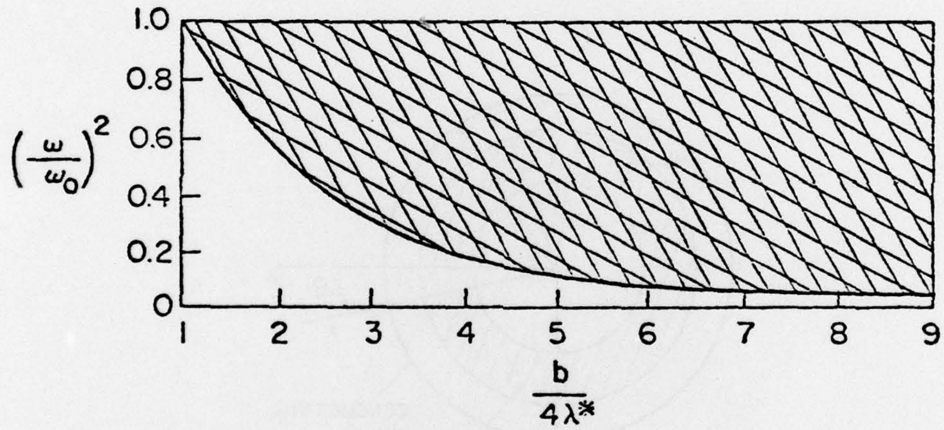

Fig. 2 — Allowed region (cross hatched) of the normalized frequency ω/ω_o as a function of $b/4\lambda^*$